AMPLIFIERS

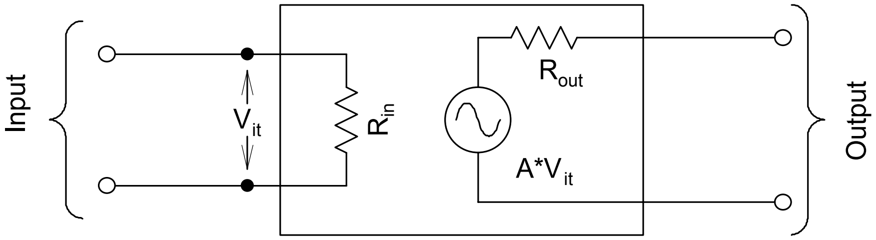
INTRODUCTION

In this section, we will introduce transistors as a new circuit element and will focus on the design of transistor amplifiers. To provide appropriate background for the discussion with transistors, we will begin with a general discussion of amplifiers.

AMPLIFIERS

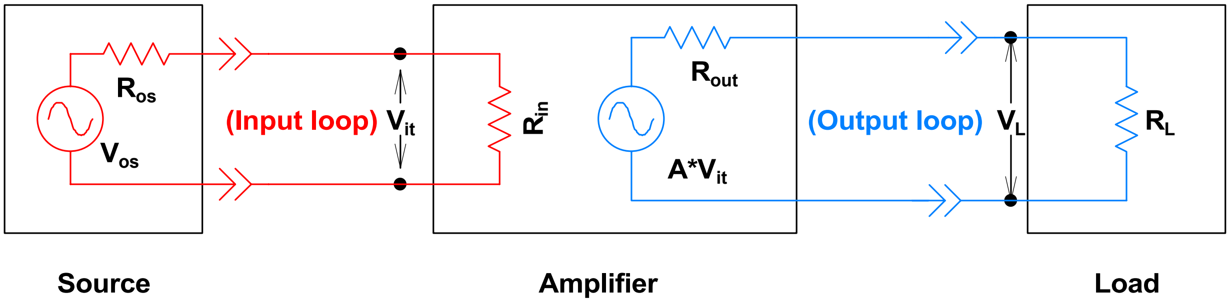
The defining behavior of an amplifier is that the output is in some sense "larger" than the input. (This is like the behavior of a magnifying glass.) Of course, we generally want the output to be a faithful replica of the input–that is, we want the amplification to be ***linear***. The linear relation between the output and input, $A = \frac{\text{out}}{\text{in}}$ defines A, the ***amplification.*** In the sense of this relation, ***linear*** means that A is a ***real constant***. The purpose of the amplifier may be to amplify ***voltage***, ***current***, or ***power***. Although our main focus will be on ***voltage amplifiers***, we will briefly consider all three types.

An operational model for an amplifier is:



In this model, R_{in} indicates the ***input resistance***, R_{out} the ***output resistance***, V_{it} the voltage at the ***input terminals***, and A^*V_{it} is the amplified version of V_{it}. This model comes directly from Thevenin's theorem: the output section of an amplifier should be a signal source and thus the Thevenin representation is an ideal voltage source in series with a resistance. In contrast, the input section is *not* a source; thus it behaves simply as a resistance. The voltage source represented in the output section is the voltage resulting from amplifying the voltage at the input terminals.

However, an amplifier is useful only if it is connected to a source supplying the signal to be amplified, and to a load where the amplified result will be "delivered." Consequently, the model for an"***amplifier system*** is:



(In the sketch, $\boldsymbol{V_{os}}$ is the open-circuit output voltage of the source, $\boldsymbol{R_{os}}$ is the output resistance of the source, and $\boldsymbol{R_L}$ is the load resistance.) With an amplifier system, the standard objective is for the voltage across the load to be as nearly equal as possible to the amplification factor times $\mathbf{V_{os}}$. Analysis of the input loop shows the requirement for maximizing V_{it}, the voltage actually amplified:

$$V_{it} = V_{os}\left\{\frac{R_{in}}{R_{os}+R_{in}}\right\} = V_{os}\left\{\frac{1}{1+R_{os}/R_{in}}\right\}$$

Clearly, V_{it} is maximum when the fraction R_{os} / R_{in} is zero. A similar analysis of the output loop yields

$$V_L = A * V_{it}\left\{\frac{R_L}{R_{out}+R_L}\right\} = A * V_{it}\left\{\frac{1}{1+R_{out}/R_L}\right\}$$

Also clearly, V_L is maximum when R_{out} / R_L is zero. Thus, from the perspective of ***amplifier design***, the objectives are to make $R_{in} \to \infty$, while making $R_{out} \to 0$.

EXAMPLE. Calculate the ***Effective Amplification*** of a system where R_{os} = 20k, R_{in} = 100k, A = 100, R_{out} = 5Ω, and R_L = 2Ω. (***Effective Amplification*** is the ratio V_L / V_{os}.)

By analysis of the input loop:

$$V_{it} = V_{os}\left\{\frac{100}{20+100}\right\} = V_{os}\left\{\frac{5}{6}\right\}, \text{and}$$

$$V_L = 100 * V_{it}\left\{\frac{2}{2+5}\right\} = V_{it}\left\{\frac{200}{7}\right\}.$$

Thus,

$$EA = \frac{V_L}{V_{os}} = \left\{\frac{5}{6}\right\}\left\{\frac{200}{7}\right\} = \frac{500}{21} = 23.8$$

The clear result is that EA < 100! This is a direct consequence of the “loading” of the source by R_{in} of the amplifier and of loading of the amplifier’s output by the load.

Considerations for Current and Power Amplifiers: The analysis above focused on the objectives in a voltage-amplifier system of maximizing “delivery” of voltage from the source to the input terminals of the amplifier, and maximizing delivery of voltage from the amplifier output to the load. The obvious question is what considerations hold for current and power amplifier systems. For current amplifiers, the solution is fairly obvious: current to the input terminals of the amplifier is maximized when R_{in} is zero, and load current is maximized when R_{out} is large. These are the exact opposites of the "desired" cases for voltage amplifiers!

For power amplifiers, the objective is to maximize delivery of the power, the product V*I. Obviously, neither $R_{in} = 0$ nor $R_{in} \to \infty$ are optimal as V is zero in one case, I in the other, and P is zero for both! Since the input and output loops are the same type of circuit, the maximization problem is the same for both. Specifically considering the input loop:

$$P_{in} = I_{in}^2 R_{in} = V_{os}^2 \left\{ \frac{R_{in}}{(R_{in} + R_{os})^2} \right\},$$

Thus the problem is to find R_{in} maximizing the quantity in braces; *i.e.,* to find R_{in} such that the derivative of the quantity in braces with respect to R_{in} equals zero:

$$\frac{d}{dR_{in}} \left\{ \frac{R_{in}}{(R_{in} + R_{os})^2} \right\} = -2 \left\{ \frac{R_{in}}{(R_{in} + R_{os})^3} \right\} + \left\{ \frac{1}{(R_{in} + R_{os})^2} \right\}$$

$$= \left\{ \frac{R_{in} + R_{os} - 2R_{os}}{(R_{in} + R_{os})^3} \right\} = \left\{ \frac{R_{in} - R_{os}}{(R_{in} + R_{os})^3} \right\} = 0$$

Thus, P_{in} is maximum when $\mathbf{R_{in} = R_{os}}$.

By the same type of analysis, P_L is maximum when $\mathbf{R_L = R_{out}}$.

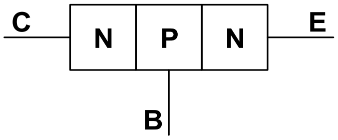
AMPLIFIER SUMMARY

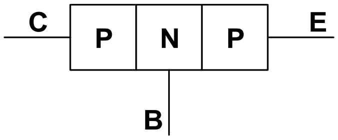
The basic result is that the amplifier presents a load to the signal source, and the delivery of signal to the load is affected by both the load resistance and the amplifier's output resistance. For a ***voltage amplifier***, the objective of maximum voltage "transfer" leads to the design targets of having the input resistance ***as high as possible*** and the output resistance ***as low as possible.***

In contrast, for a ***current amplifier,*** the objective of maximum current "transfer" creates the goals that the input resistance should be ***as low as possible*** and the output resistance should be ***as high as possible***. Finally, for ***power amplifiers***, the objective of maximum power transfer creates the goals that the input resistance should ***equal that of the source***, and the output resistance should ***equal that of the load***.

TRANSISTORS AND TRANSISTOR AMPLIFIER DESIGN

As indicated above, transistors are the new circuit element introduced in this section. Transistors are related to diodes discussed previously in that they incorporate 3 sections of P- and N-doped semiconductors as shown below:

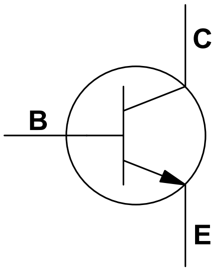




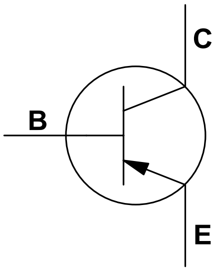
Also as indicated, there are two possible configurations, "NPN", and "PNP." ***Junction bipolar*** transistors are therefore classified with this terminology. Moreover, the connections are designated as "collector," "base," and "emitter," or ***C, B,*** and ***E***, respectively. (From the simplistic sketches shown, it is not clear that C and E are different in any way; however, they are quite different in the actual construction of a transistor.)

Also, two PN-junctions are apparent in each arrangement: base-emitter and collector-base. At the design level we will use, the base-emitter junction is the important one in that it will be necessary to remember that current flow through a PN junction requires that the forward threshold voltage be established (*e.g.,* V_f = ~ 0.7V for Silicon).

Circuit symbols for each type of (junction bipolar) transistor are:



NPN

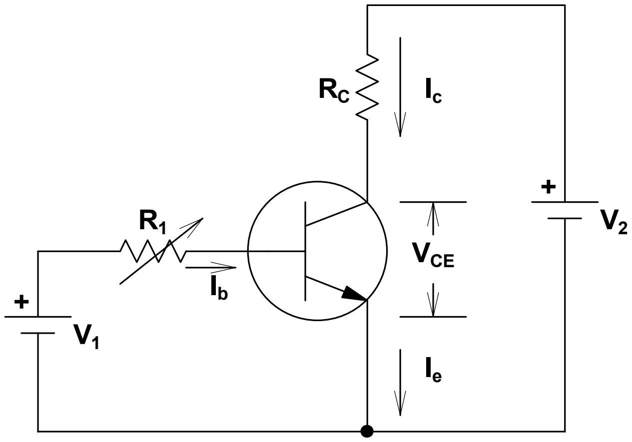


PNP

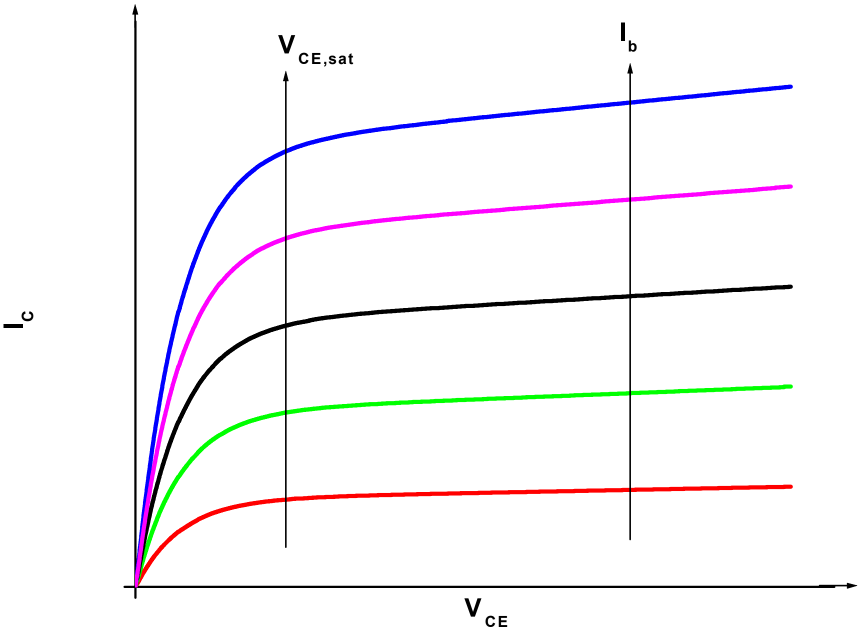
Our interest is in learning how to use the electrical characteristics of transistors to accomplish the function of amplification. For this reason, we will focus only on the NPN type; thus, our ***typical transistor*** will be an ***NPN*** version constructed ***of silicon***. (For our purposes, the basic difference between NPN and PNP behavior is that current flow in one is in the opposite direction from the other. Thus all statements made below regarding NPN transistors also apply to PNP versions when the current directions are reversed.)

Electrical Behavior of Transistors: The basic circuit sketched below indicates the main parameters of interest to us: the base current, I_b; the collector current, I_c; and the collector-emitter voltage, V_{CE}. The graph shown below the circuit illustrates the electrical relationships between these parameters as often presented in textbooks. Apparent from the graph is the characteristic that the collector current is virtually proportional to the base current at *constant* V_{CE}, and is pretty much proportional regardless of V_{CE} as long as it is above the value indicated as $V_{CE,sat}$.

(It is important to note here that many textbooks build their amplifier design procedure on the set of curves shown in the graph. However, this procedure, a carry-over from vacuum-tube-based amplifier design methods, is inappropriate for transistor amplifier design for at least two reasons. One is that transistor manufacturers do not provide such "characteristic curves" for their products, and the second is that manufacturing processes are such that the characteristics of the same transistor type number may vary over a wide range.)



"Typical" Transistor Characteristic Curves



Based on the characteristic curves, and for practical purposes, we can take the ***basic property*** of transistors as ***"the base current CONTROLS the collector current."*** In addition, the collector current is larger than the base current by a factor of 30 to 300; thus changing the base current leads to a corresponding change in a much larger current. In this sense, transistors are inherently ***current amplifiers***. (It is important to point out that the base current does not *create* collector current; it only exercises control over the collector current. It other words, the base is like a faucet: the faucet can control the flow of water, but it cannot create water when none is present.)

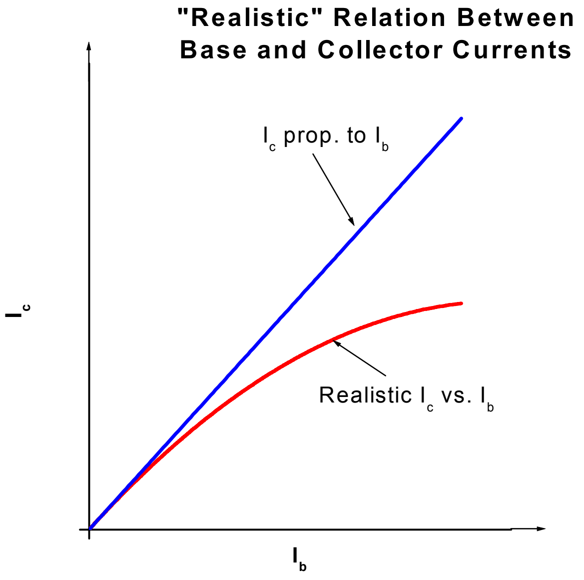
Along with the basic behavior that I_b controls I_c, is the fact that their relationship is approximately proportional. (Of course, this is not ***exactly*** true although a detailed description of transistor parameters is more involved than necessary for our purposes.)

Specifically, we will make use of the following relations between base and collector currents:

$$h_{FE} = \frac{I_C}{I_B} \text{ ("large-signal" current gain; very crude approximation, but useful)}$$

$$h_{fe} = \frac{\Delta I_C}{\Delta I_B} \text{ ("small-signal" current gain; better approximation)}$$

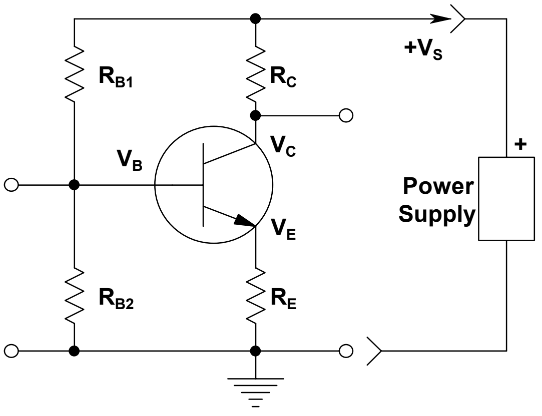
Sketched to the side is a reasonable representation of the actual relationship between I_C and I_B.



Summary of Our "Typical" Transistor Characteristics:

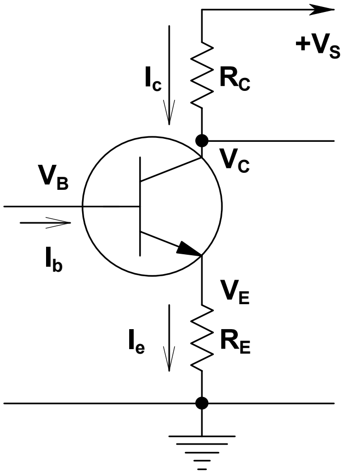
1. Type: NPN
2. Material: Silicon (V_{BE} = 0.7 V for $I_B \neq 0$)
3. $h_{fe} = h_{FE}$ ~100

Amplifier Design. As indicated above, the basic nature of a transistor is that of a "current amplifier." Our objective is to use this character to create a voltage amplifier leading to the obvious question of how we can "convert current into voltage." The method, of course is to use resistors as "current-to-voltage converters." Therefore, our "standard" transistor amplifier circuit will be as sketched below:



(The power supply and its connections will be omitted from this point on. Thus, it is understood that amplifiers require a power supply, and that the power supply + will be connected to the point labeled "+V_S" and with - connected to ground, or "common.")

Voltage Relations for Transistors: Before the discussion of amplifier design, it will be helpful to think through voltage relationships appropriate for a circuit such as that of the standard amplifier. Specifically, consider the fragment of the amplifier circuit consisting of the transistor along with the collector and emitter resistors. We are only interested in the case of non-zero base current; in that case the following relations hold:



1. $V_B = V_E + 0.7$ ($V_{BE} = V_F$)
2. $I_E = V_E / R_E$ (Ohm's Law)
3a. $I_E = I_B + I_C$ (KCL)
3b. $I_C = h_{fe} {}^* I_B$ (Transistor Property)
3c. $I_E = I_B {}^* (h_{fe} + 1)$ (*3a.* and *3b.*)
3d. $I_E \sim I_B {}^* h_{fe} = I_C$ ($h_{fe} \sim 100$; can ignore difference between 100 & 101))
4. $V_C = V_S - I_C {}^* R_C$ (Ohm's Law)

EXAMPLE:

Given the following data for the circuit "fragment" sketched above, calculate the voltages at the emitter and collector and the currents indicated on the sketch. ***(V_B = 1.1V; R_E = 250Ω; R_C = 3kΩ, V_S = 15V.)***

Solution:
$\mathbf{V_E}$ = (1.1 - 0.7)V = **0.4 V**
$\mathbf{I_E}$ = 0.4V / 250Ω = **1.6 ma**
$\mathbf{V_C}$ = 15V - (1.6ma) * (3kΩ) = **10.2V**
$\mathbf{I_C}$ ~ I_E = **1.6 ma**
$\mathbf{I_B}$ ~ I_C / h_{fe} = 1.6ma / 100 = **16 μa**

Basic Point: Just as the base current controls the collector current (and emitter current), so does the base voltage control the collector and emitter voltages.

EXAMPLE:

For the resistors and V_S the same as in the example above, calculate the base voltage and current necessary to make the collector voltage = 7.5V.

Solution:
$\mathbf{I_C}$ = (15 - 7.5)V / 3kΩ = **2.5 ma**
$\mathbf{V_E}$ ~ 2.5ma * (0.25kΩ) = **0.625V**
$\mathbf{V_B}$ = 0.625V + 0.7V = **1.325V**
$\mathbf{I_B}$ ~ I_C / h_{fe} = 2.5ma / 100 = **25 μa**

Amplifying

We can now analyze the circuit fragment as an amplifier; in fact, this portion of the "standard" circuit is responsible for the amplification. The basic idea is to establish a base voltage, and therefore a collector voltage, as a steady value. "Amplification" will amount to the ***change in the collector voltage*** (the output) caused by ***changing the base voltage*** (the input). For this circuit, amplification will be expressed as $A = \frac{V_{out}}{V_{in}} = \frac{\Delta V_C}{\Delta V_B}$. By use of the relations developed above, we can relate amplification to the circuit elements as follows:

$$I_C \sim I_E = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E}$$

$$V_C = V_S - I_C R_C = V_S - \left(\frac{V_B - 0.7}{R_E}\right) R_C$$

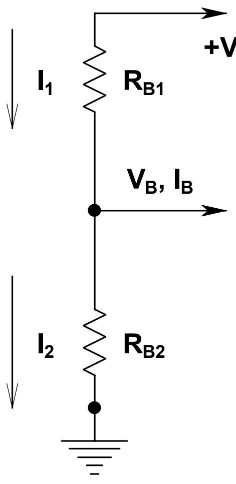
Since $V_S, 0.7, R_C,$ and R_E are constants,

$$A = \frac{\Delta V_C}{\Delta V_B} = -\frac{R_C}{R_E} .$$

Thus the amplification is controlled by only the collector and emitter resistors with no explicit dependence on the transistor parameters! This of course raises the question of why the transistor is necessary. However, the transistor plays a significant role since it "makes" the collector and emitter voltage different by a constant amount; it makes the emitter and collector currents virtually equal; and thus it makes the collector voltage respond to the base voltage as described. (The "-" in the expression for amplification is the result of taking the output voltage from the collector to ground: as I_C increases, this voltage decreases.)

Role of Base Resistors R_{B1} and R_{B2}:

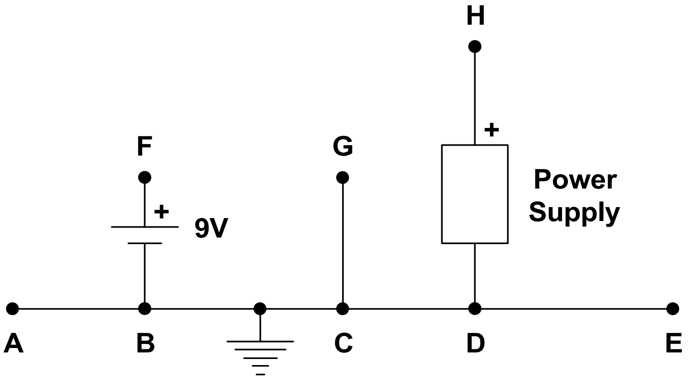
As seen above, the amplification is controlled by only R_C and R_E. However, to have a working amplifier circuit, it is necessary that the base voltage and current be "suitably" established, and this is the purpose of the base resistors. Specifically, the resistors must be suitable for setting up the electrical situation sketched to the side. However, since the basic circuit is that of a voltage divider and there are two resistors, the electrical requirements can be met in an infinite number of ways as they require only that the resistors be related appropriately. This situation, one of "underdetermined parameters" gives us the opportunity to incorporate an additional design target into the overall amplifier design.



In fact, before continuing with the discussion of methods for selecting "suitable" values for the resistors, let us note that the amplification relation also does not uniquely determine the collector and emitter resistors. Since neither the amplification nor the electrical requirements at the base yield unique values for the set of 4 resistors, it will be useful to look for other desirable amplifier characteristics we may include in and meet with the overall design considerations. In connection with amplifiers in general we already discussed above two such characteristics: the ***input resistance*** and the ***output resistance.*** To see how target values for these characteristics can be included in the design, we need to analyze the "standard" amplifier circuit to learn the factors on which these depend.

The Concept of "ac Ground":

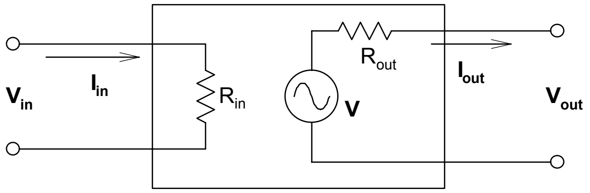
To obtain the relations necessary for the equivalent input and output resistances of our amplifier, we need to introduce the concept of "ac ground." The basic ideas are : (1) the difference between ac and dc is that ***ac changes***; and (2) that "ground" is the set of connections used as the "0-volt" reference for all measurements. (Ground also is one connection for applying input signals, and one for obtaining outputs; as well it often is one connection to the power supply.) Since "dc ground" is the set of all points at voltages ***no different from*** ground, "ac ground" is the set of all points which ***do not change relative to*** ground. For the discussion consider the sketch: in it, the points A, B, C, D, E, and G are all connected directly to ground and thus are at dc ground; that is, they are ***at a voltage equal to dc ground***. Similarly, points F and H ***are not*** dc ground, but ***are*** ac ground as they are connected to ground by a constant voltage. In summary, all points in the sketch are ***ac ground.***



Input and Output Resistance for our "Standard" Transistor Amplifier:

The approach for relating the amplifier's input and output resistance to the actual circuit is based on Thevenin's Theorem. Specifically, referring to the general amplifier model as introduced above (see sketch below), the ***input resistance*** can be calculated by developing the relation between and input voltage and the current flowing into the amplifier, while the ***output resistance*** can be obtained by relating the open-circuit out put voltage to the short-circuit output current. These are summarized below:

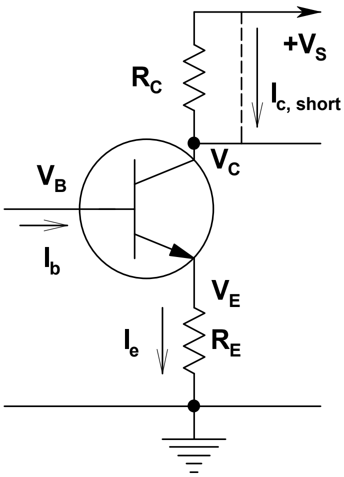
$$R_{in} = \frac{V_{in}}{I_{in}}$$



$$R_{out} = \frac{[V_{out}]_{open}}{[I_{out}]_{short}}$$

Output Resistance:

Referring to the amplifying fragment shown to the side and recalling that the "output" is the *change* in the collector voltage, we can identify that $(V_{out})_{open} = \Delta V_C$. The direct approach to calculating the output short-circuit current is to short the collector to ground. However, this complicates the calculation; the alternative is to recall the "ac ground" concept and make the short circuit from collector to V_S as indicated by the dotted line. From this it is clear that $(I_{out})_{short} = \Delta I_C$; thus



$$R_{out} = \frac{[V_{out}]_{open}}{[I_{out}]_{short}} = \frac{\Delta V_C}{\Delta I_C} = \mathbf{R_C}$$

In summary, therefore, the amplifier's ***output resistance*** is determined by (and is equal to!) The collector resistor. (Of course, the *exact* story is more complicated; nevertheless, this result gives the dominant contribution to R_{out}.)

Input Resistance:
Obtaining the expression for the input resistance requires a two-step approach: first we will consider the "basic amplifying section" sketched above, and then we will consider the effect of the base resistors. Referring to the sketch above and the relations developed earlier for the "input" of $\Delta V_{B:}$

$$R_{in} = \frac{\Delta V_B}{\Delta I_B}$$

$$\Delta V_B = \Delta(V_E + 0.7) = \Delta V_E = \Delta I_E R_E$$

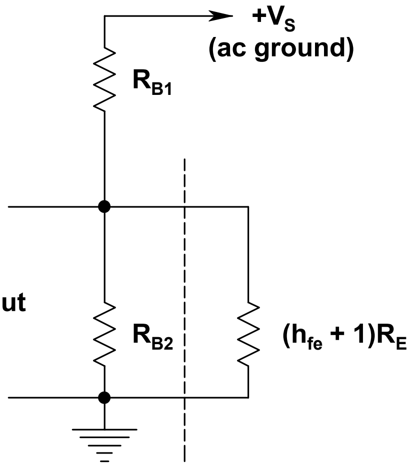
$$\Delta I_E = \Delta(I_B + I_C) = (h_{fe} + 1)\Delta I_B$$

$$\Delta V_B = \Delta I_E R_E = (h_{fe} + 1)\Delta I_B R_E$$

$$R_{in} = (h_{fe} + 1)R_E$$

This result describes only the "basic amplifying section" and now must be combined with the base resistors to calculate the effective input resistance of the amplifier under operating conditions. The circuit to consider is as sketched below:

From the sketch, it is clear that R_{B2} is parallel to the effective input resistance of the "basic amplifying section", $(h_{fe} + 1)R_E$. The effect of R_{B1} can be included by recalling that the input signal is a changing voltage, and thus is an ac signal. For signals, therefore, V_S is "ground" since it is a point of ac ground. Thus, for the input signal, the effective input resistance is the equivalent of R_{B1} // R_{B2} // $(h_{fe} + 1)R_E$.



Summary of R_{in} and R_{out} Relations:
For our transistor amplifier circuit, the effective input resistance (for signals) is:

$$\mathbf{R_{in} = R_{B1} \,//\, R_{B2} \,//\, (h_{fe} + 1)R_E}$$

and the effective output resistance is:

$$\mathbf{R_{out} = R_C.}$$

Methods for Calculating "Suitable" Resistor Values for the Amplifier Circuit:
For our purposes, the "standard" problem can be summarized as: given a "typical" transistor (NPN silicon) and a power supply of voltage V_S, find "suitable" resistors to meet the specifications: ***$A \geq$ "specified value"; $R_{out} \leq$"specified value"; $R_{in} \geq$"specified value."***

Example and Illustration of (One) Procedure

Given V_S = 24V and a “typical” transistor, find resistor values to meet the following specifications: $|A| \geq 15$, $R_{out} \leq 3k$, and $R_{in} \geq 12k$.

Solution:

$$\left.\begin{array}{l} |A| \geq 15 \rightarrow \text{CHOOSE } |A| = 15 = R_C / R_E \\ R_{out} \leq 3k \rightarrow \text{CHOOSE } R_{out} = 3k \end{array}\right\} \begin{array}{l} R_C = 3k \\ R_E = 200\Omega \end{array}$$

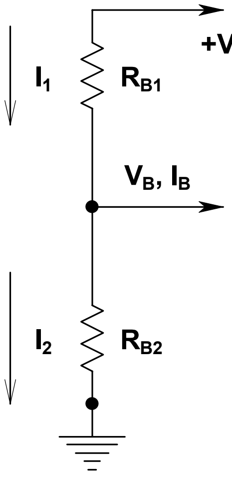
Good Idea: The input and output signals are ac voltages, and the basic operation of the circuit is that input changes V_B which leads to a change in V_C as the “output” voltage. Thus, it makes sense to ***aim*** for an initial voltage at the collector maximizing the ***upwards*** and ***downwards*** change in collector voltage. Obviously, the best starting point is $V_C = \frac{1}{2} V_S$.

AIM for $V_C = \frac{1}{2} V_S = 12V$; $I_C R_C = V_S - V_C = 12V$; $I_C = (V_S - V_C) / R_C = 12V / 3k = 4ma$;

The goal now is to find out what V_B and I_B are necessary for the target V_C:

$V_E = I_E R_E \sim I_C R_E = (4ma)(0.2k) = 0.8V$; $V_B = V_E + 0.7V = (0.8 + 0.7)V = 1.5V$
$I_B \sim I_C / h_{fe} = 4ma / 100 = 40\ \mu a$;

Base R’s: To meet the R_{in} specification and provide the necessary electrical conditions, R_{B1} and R_{B2} must create V_B = 1.5V, provide I_B = 40 μa, and be such that $R_{B1} // R_{B2} // (h_{fe} + 1) R_E$. An effective way to proceed is based on ***educated guessing***. Specifically, $(h_{fe} + 1)R_E$ is already determined; moreover, R_{B2} is (usually) the smaller of the two base resistors since it has less voltage across it. Since the equivalent of a parallel arrangement of resistors is less than the smallest member, R_{B2} will play this role.



For the example, $(h_{fe} + 1) R_E = 100 (0.2k) = 20k$; to make $R_{in} \geq 12k$, the equivalent of $R_{B1} // R_{B2}$ must be greater than or equal to 30k. Therefore, R_{B2} must itself be more than 30k; so **TRY R_{B2} = 40k**.

$I_2 = V_B / R_{B2} = 1.5V / 40k = 37.5\mu a$
$I_1 = I_B + I_2 = 77.5\ \mu a$
$\mathbf{R_{B1}} = (V_S - V_B) / I_1 = 22.5V / 77.5\mu a \sim$ **290 k**

Check: Use of a trial-and-error procedure requires that the result be checked to see if it meets the specifications:

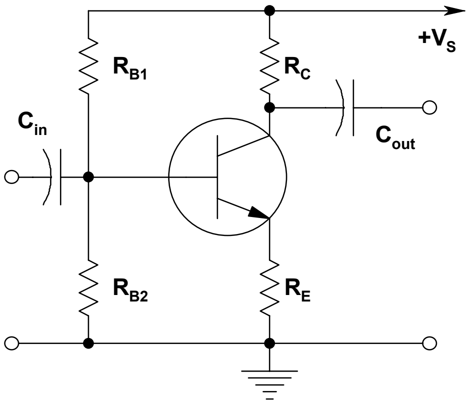
$R_{in} = R_{B1} // R_{B2} // (h_{fe} + 1)R_E$ = **290k // 40k // 20k = 12.74K > 12k, so OK.**

Input and Output Coupling Capacitors:
Since the input point (the base) and the output point (the collector) are at carefully established dc voltages, the input and output (ac voltages) need to be coupled via capacitors. These serve as "DC-blocking" elements, which is another way of saying they create high-pass filters in the overall circuit. We need to consider two points in this regard:

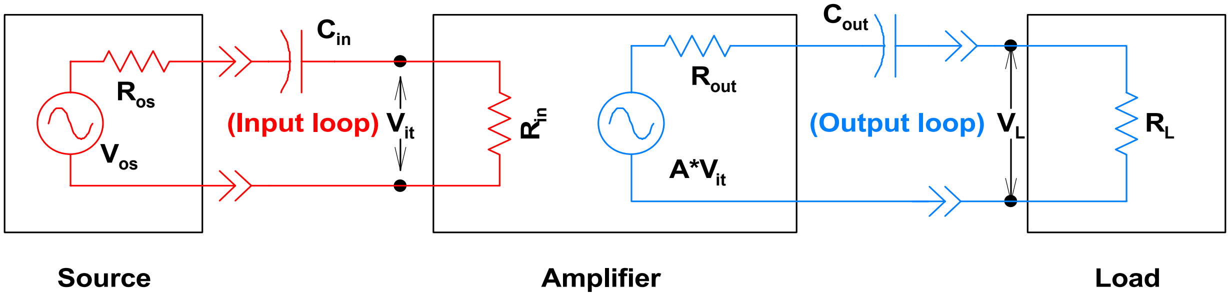
(1) what are appropriate considerations for selecting suitable values for C_{in} and C_{out}?

(2) how do these capacitors affect the overall performance of the amplifier?

Sketched below is the "standard" amplifier including the coupling capacitors:



These create the effective circuit as modeled below:



The basic effect is incorporation of the capacitors in the input and output loops. A reasonable "rule-of-thumb" is to calculate capacitor values so that their respective reactances equal the amplifier's input resistance and the load resistance at a chosen low frequency.

EXAMPLE:

Calculate "suitable" input and output coupling capacitors for an amplifier with R_{in} = 10k to be connected to a load R_L = 100Ω at the target low frequency (2Hz/π).

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} \Rightarrow C_{in} = \frac{1}{2\pi\left(2/\pi\right)10k} = 25\mu F$$

$$C_{out} = \frac{1}{2\pi\left(2/\pi\right)100} = 2500\mu F$$

Finally, the complete description of the loading including the capacitors requires treating the input and output loops as ac circuits:

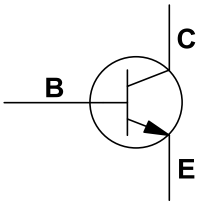
$$V_{it} = R_{in}\left(\frac{V_{os}}{R_{os} + R_{in} + - j/\omega C_{out}}\right)$$

$$V_L = R_L\left(\frac{AV_{it}}{R_{out} + R_L - j/\omega C_{in}}\right)$$

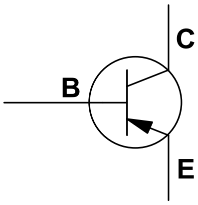
$$EA = \frac{V_L}{V_{os}} = \left(\frac{AR_L}{R_{out} + R_L - j/\omega C_{out}}\right)\left(\frac{R_{in}}{R_{os} + R_{in} - j/\omega C_{in}}\right)$$

$$|EA| = \left|\frac{V_L}{V_{os}}\right| = \left(\frac{AR_L}{\sqrt{(R_{out} + R_L)^2 + \left(1/\omega C_{out}\right)^2}}\right)\left(\frac{R_{in}}{\sqrt{(R_{os} + R_{in})^2 + \left(1/\omega C_{in}\right)^2}}\right)$$

GLOSSARY



NPN Transistor



PNP Transistor