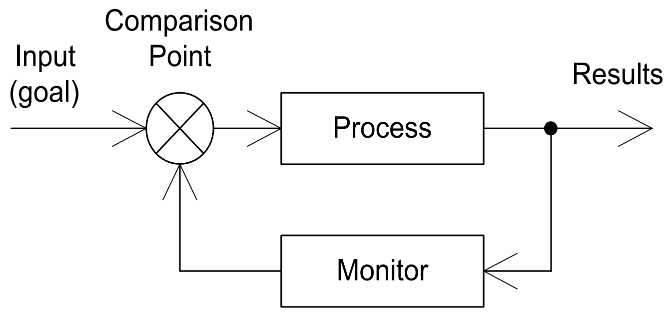
FEEDBACK

INTRODUCTION

The basic idea of feedback is straightforward: the results of some process are monitored, compared to a goal, and the input is adjusted based on the difference between the results and goal. This can be represented diagrammatically as follows:



Generalized diagram of Feedback

In fact, in everyday life, we can identify many examples of feedback: keeping our car on the road when driving, behavior modification techniques, *etc.* Of course, we will restrict our attention to electronic implementation of feedback. From that perspective, the signals to monitor and control may be voltage, current, or power. As we have done previously, we will further restrict ourselves to cases of voltage feedback.

Amplifier-based voltage-feedback systems are basically as indicated below. (The set of +'s and -'s indicate the standard sign convention for summing voltages in the input loop.) From the diagram, we can derive the relation between the input and output, also as indicated below.

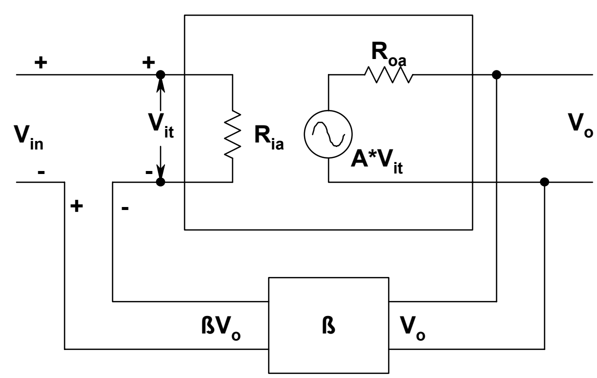
Feedback Amplifier Analysis:

$V_{it} = V_{in} + \beta V_o$ (KVL input loop analysis)

$V_o = AV_{it} = A(V_{in} + \beta V_o)$ (Based on no-load output)

$$\frac{V_o}{V_{in}} = \frac{A}{1-\beta A} = G$$ (Voltage-feedback amplifier relation)

Voltage Feedback Amplifier System



DISCUSSION OF THE FEEDBACK RELATION

Prior to making use of feedback, we need to examine the feedback relation for G in some detail. First, the factor represented by β can be many things: another amplifier, a filter, a phase-shift network, a voltage divider, *etc.* However, we will represent its effect as the mathematical factor ***β which in general is a complex number***. Likewise, the amplifier may introduce phase-shift, and / or frequency dependence; thus ***A in general also is a complex number***. ***The product Aβ***, appearing in the denominator of the feedback relation ***is therefore in general a complex number***.

From our previous discussion of complex number algebra, we know that the magnitude of a product is the product of the magnitudes, and that the phase angle of a product is the sum of the phase angles. Thus, for emphasis, we can express Aβ as $|A\beta| = |A||\beta|$ **and** $\varphi_{A\beta} = \varphi_A + \varphi_\beta$. Recall also that inversion (multiplication by -1) is the same as a 180° phase shift, and that 0°, 360°, ...Nx360° all describe the in-phase condition. From this, we can identify two basic cases of the expression for G to examine further:

1. $A\beta > 0$ (Positive real number)

means $\phi_A + \phi_\beta = 0^\circ, 360^\circ, ..., n * 360^\circ$

Positive Feedback

2. $A\beta < 0$ (Negative real number)

means $\phi_A + \phi_\beta = 180^\circ, 540^\circ, \ ...180^\circ + n * 360^\circ$

Negative Feedback

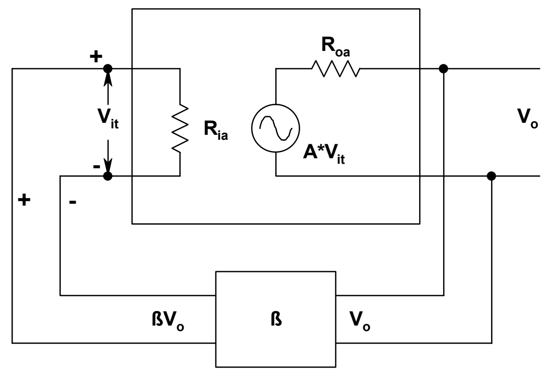
Of course, there are in-between cases where Aβ is a complex number. However, these two cases listed above, where the Aβ product is a real number, are very inportant for several reasons, some of which will be discussed below.

Positive Feedback. Positive feedback is the situation where the adjustment is applied to reinforce (or add to) the difference between the goal and the results. Obviously. This adjustment moves the results even farther from the goal and is undesirable in most cases. (In psychology and behavior modification, the term "positive feedback" is often used; however, it matches our description only from the view that the feedback moves the results further away from an undesirable goal.) For example, in driving a car, one aim is to keep the car along the chosen path. This is accomplished by monitoring the actual path in comparison to that desired and applying an adjustment ***opposite*** to the ***difference***. If the adjustment reinforced the difference, as would be the case with positive feedback, the path would become even more in error—perhaps leading to the ditch!

Nevertheless, positive feedback in electronics has at least one clear application, that of producing oscillators. For example, consider the case Aβ = 1, which makes the denominator 1 - Aβ = 0, and G infinite! A physical interpretation of this condition is that β represents signal reduction in the feedback path while A represents restoration. That is, if A exactly restores that which was lost, then the feedback creates a self-sustaining signal (as long as it is started in the first place). If A > 1/β, the signal grows until limited by some other factor (power supply, etc.). In the case of $A\beta \geq 1$, no "input" in needed and the feedback amplifier circuit becomes as shown in the circuit below.

Notice that $V_{it} = \beta V_o$ and that the algebra seems contradictory if we write $V_o = A\beta V_o$ when $A\beta > 1$.

Voltage Feedback Amplifier System, Oscillator Configuration

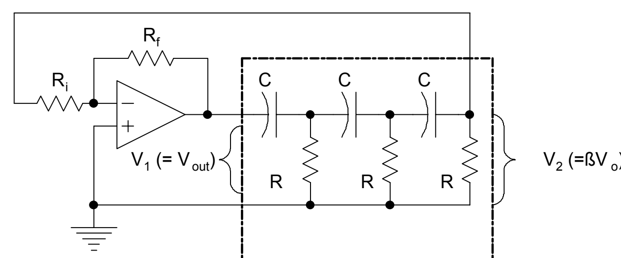


The condition for self-sustaining signals (or oscillation, called the ***Barkhausen Criterion***, is

$$A\beta = 1,$$
$$\text{or } |A||\beta| = 1,$$
$$\text{and } \phi_A + \phi_\beta = 0^\circ, 360^\circ, ..., n * 360^\circ.$$

For oscillators, a main application of positive feedback, β is designed to have a frequency dependence so that $A\beta \to 1$ @ f_o. For example, consider the phase-shift oscillator sketched below. The amplifier is in the op-amp inverting configuration. Therefore, ϕ_A = 180° and at f_o it is also necessary that ϕ_β = 180°. An a.c. circuit analysis of the RC network (straightforward, but tedious) yields:

Phase Shift Oscillator



$$\beta = \frac{V_2}{V_1} = \frac{1}{1 - \frac{5}{(\omega RC)^2} + j\left[\frac{1}{(\omega RC)^2} - 6\right]\frac{1}{\omega RC}}$$

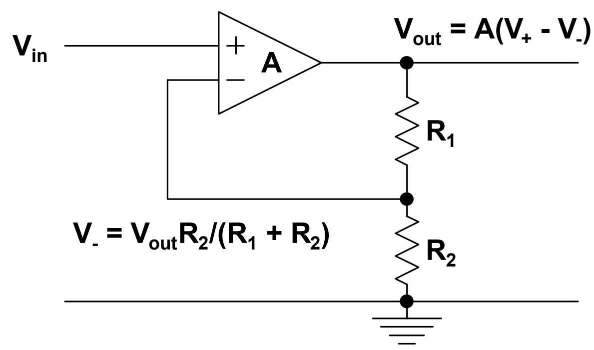
V_o / V_i is purely real when the imaginary term in square brackets is zero, or when $(\omega RC)^2 = 1/6$. Also, when this condition is satisfied, $\beta = \frac{V_o}{V_i} = \frac{1}{1 - 5\cdot 6 + j[0]} = -\frac{1}{29}$. Thus, for the phase-shift oscillator shown above, the Barkhausen criterion is satisfied at $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{6}RC}$ if $A \le -29$ ($|A| \ge 29$).

Negative Feedback. Negative feedback is the case more generally applied. For the example of driving described above, adjustment of the car's direction opposite to the deviation ***corrects the deviation*** and is an example of negative feedback. Many applications of negative feedback are in control-system design (guidance systems, voltage regulators, *etc.*) However, because the phase shift of electronic (and mechanical devices) is a function of frequency, usually increasing with increasing frequency, it is necessary that the design prevent unintentional occurrence of the Barkhausen condition. (You wouldn't like being a passenger on a plane where the autopilot exhibited this condition!) In fact, the central problem of control-system design is to provide quick response while preventing occurrence of the Barkhausen condition.

Several circuits we have already considered employ negative feedback. Of these, we will examine two in more detail, the non-inverting amplifier configuration with operational amplifiers and our transistor amplifier. In addition, we will examine a voltage regulator system using negative feedback.

Negative Feedback and the Non-inverting Op-Amp Circuit. Shown below is a re-sketch of this amplifying circuit emphasizing the connection between the output and the input. The main new element in the analysis is retaining A as a finite value. (R_{in} is still taken as large enough to make $I_{amp} \sim 0$.) Analysis of the circuit yields:

Non-Inverting Op-Amp



$$V_+ = V_{in}$$

$$V_- = V_{out}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{out} = A(V_+ - V_-) = A\left(V_{in} - \frac{V_{out}R_2}{R_1 + R_2}\right)$$

$$\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{\mathbf{A}}{\mathbf{1 + AR_2/(R_1 + R_2)}} = \mathbf{G}$$

Since this expression is different from that obtained previously, but the previous derivation used the approximation $A \to \infty$, we should examine this new result for correspondence to the earlier one. Doing so gives the expressions:

$$\lim_{A \to \infty}\left(\frac{V_{out}}{V_{in}}\right) = \lim_{A \to \infty}\left(\frac{A}{1 + AR_2/(R_1 + R_2)}\right) = \lim_{A \to \infty}\left(\frac{1}{\frac{1}{A} + R_2/(R_1 + R_2)}\right) = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

Things are OK since this is the same result as before. Also, by examination of the expression for G, it is evident that $A\beta = -A\frac{R_2}{R_1 + R_2}$ for this circuit.

Example 1: Calculate the ***actual*** voltage "Gain" of a circuit in the non-inverting op-amp configuration when ***A = 50*** and ***R_1 / R_2 = 99.***

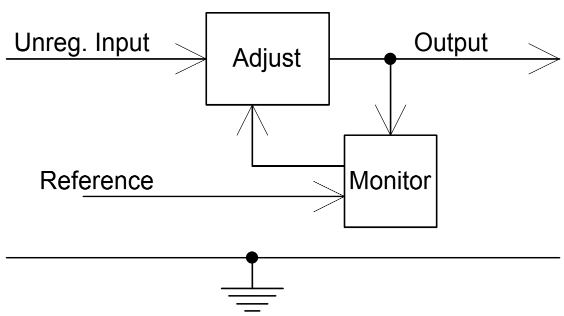
Solution: Using the result above, we get $G = \frac{50}{1 + 50\frac{R_2}{R_2 + 99R_2}} = \frac{50}{1 + \frac{50}{100}} = 33\frac{1}{3}$.

Example 2: Calculate the minimum value of A to make the ***actual*** voltage "Gain" of a circuit in the non-inverting op-amp configuration at least 90% of the "ideal value" when ***R_1 / R_2 = 99.***

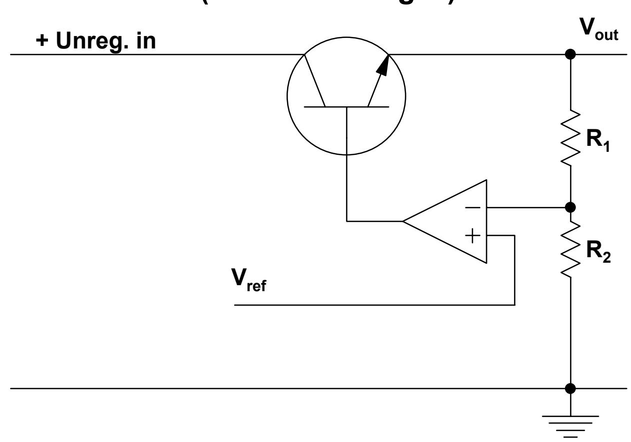
Solution: The "ideal" value is $\mathbf{G = 1 + R_1 / R_2 = 1 + 99 = 100.}$ So, the question becomes that of finding the value of A making $G \geq 90$. By algebraic rearrangement, we can solve (or re-derive) the expression above for A: $A = \dfrac{G}{1 - G\dfrac{R_2}{R_1 + R_2}}$. Thus the value needed is $A \geq 90 / (1 - 0.9) = 900$.

Feedback Voltage Regulator: Using feedback to accomplish the voltage regulation function can be envisioned as in the diagram. The concepts are a monitor of the actual output which compares it to the reference (desired behavior) and controls the "adjust" element. An actual implementation of such a circuit (for positive voltages) using an amplifier and an NPN transistor is shown beside the diagram.

Conceptual Feedback Voltage Regulator



Feedback Voltage Regulator (Positive Voltages)



$$V_+ = V_{ref}$$

$$V_- = V_{out}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{oa} = V_b = A(V_+ - V_-)$$

$$V_{out} = V_b - 0.7 = A(V_+ - V_-) - 0.7$$

$$V_{out} = A\left(V_{ref} - \frac{V_{out}R_2}{R_1 + R_2}\right) - 0.7$$

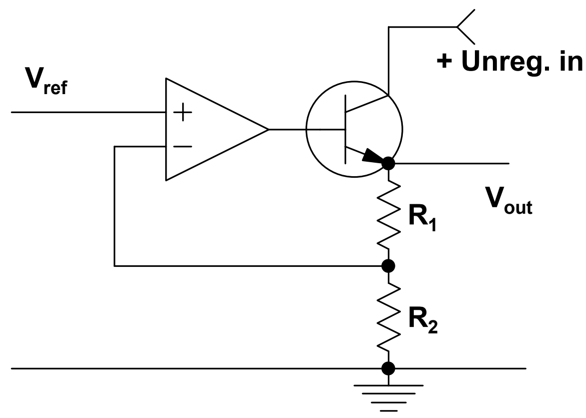
$$V_{out} = V_{ref}\left(\frac{A}{1 + A\dfrac{R_2}{R_1 + R_2}}\right) - \left(\frac{0.7}{1 + A\dfrac{R_2}{R_1 + R_2}}\right)$$

Analysis of the circuit is as follows:
The second term in the analysis, resulting from the base-emitter voltage of the transistor, is insignificant in most cases. Specifically, the denominator is the same in both terms; the numerator of the first is $_{Avref}$, while that of the second is 0.7. In cases where A is large, the first will clearly be dominant. Therefore, except for testing to see how important the second term might be, we will consider only the first term.

One other aspect of the analysis is the similarity with the result obtained above for the non-inverting op-amp configuration. Actually, this is not accidental since the voltage regulator circuit can be rearranged as shown below to emphasize its close relationship to the non-inverting amplifier circuit.

In fact, the voltage regulator is the non-inverting amplifier with a transistor in the output. In addition, V_{ref} is simply the "input." This correspondence provides another insight we will use later.

Voltage Regulator Related to Non-Inverting Op-Amp



Returning now to the voltage regulator analysis above, we see that the main term in the expression is

$$V_{out} = V_{ref}\left(\frac{A}{1+A\dfrac{R_2}{R_1+R_2}}\right)$$

which is the same as that for the non-inverting amplifier. By the same algebra, for large A, the result for the output is simply $V_{out} = V_{ref}\,(1 + R_2 / R_1)$. In other words, the output is simply the "amplified" version of the reference.

Finally, it is important to point out that the role of the reference voltage is to provide the definition of "steady." It is not necessary that the reference voltage be equal to that sought by the regulator since the output is multiplied by the factor determined by the resistors as described above. The purpose of the regulator is to provide an output with voltage steady as possible, but it can do so only with by comparison with a standard, or reference, defining "steady." This is a role played often by zener diodes in electronic voltage regulators.

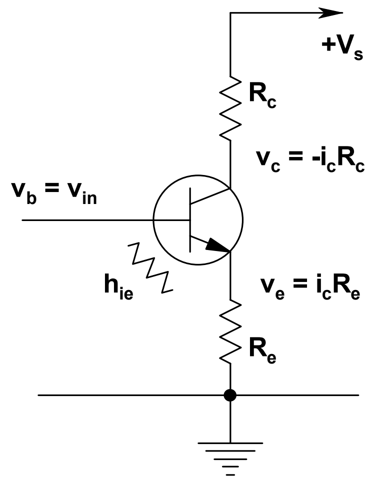
Example 3: Calculate the error in ignoring the "second" term and the actual value of A in the voltage regulator analysis for the case ***A = 25, V_{ref} = 1.25V***, and ***R_1 / R_2 = 9.***

Solution: The "ideal" value is **$V_o = V_{ref}(1 + R_1 / R_2) = 12.5V$.** The actual value using the numbers given is **V_o = 1.25 x25 / (1 + 25/10) - 0.7 / (1 +25 / 10) = 8.9V.** So the error is ~ ***3.6 V***, or about ***29%***. In fact, increasing A to 250 yields V_{out} ~ 12v, reducing the error to a much smaller amount.

Negative Feedback in Our Transistor Amplifier. As you might suspect by now, the very simple relation derived for amplification of our simple transistor amplifier circuit suggests the presence of some effect such as feedback. In fact negative feedback plays a direct and important role in the circuit. We can see this by revisiting the amplifying portion of the circuit as sketched below. The one thing we need to modify is the absolutely constant difference between base and emitter voltages. We knew this was not strictly true earlier, but must make use of the fact that the base-emitter junction can be better approximated as a resistance ***h_{ie}*** (nominal values are ~1k). Thus, while the base-emitter junction is conducting, $V_{be} \sim h_{ie} I_b$, and V_{be} is not *exactly* constant

In the circuit, the voltages expressed in lower case are to be understood as the changing components only. Recall that the circuit is an a.c. amplifier only; thus only voltage changes are important.

Transistor Amplifier for Feedback Identification



Analysis of the circuit is as follows:

$$v_{in} = v_b$$

$$v_o = v_c = -i_c\,R_c$$

$$v_e = i_c\,R_e = -\frac{v_oR_e}{R_c}$$

$$i_c = h_{fe}i_b = h_{fe}\,\frac{v_b - v_e}{h_{ie}}$$

$$v_o = -R_ch_{fe}\,\frac{v_b - v_e}{h_{ie}} = -R_c\,\frac{h_{fe}}{h_{ie}}\left(v_{in} + \frac{v_oR_e}{R_c}\right)$$

$$A = \frac{v_o}{v_{in}} = \frac{\left(-R_c\,\frac{h_{fe}}{h_{ie}}\right)}{\left(1 + \frac{h_{fe}R_e}{h_{ie}}\right)} = \frac{(-R_ch_{fe})}{(h_{ie} + h_{fe}R_e)}$$

The positive sign of the second term in the denominator indicates the negative feedback. Our approach actually used the approximation $h_{ie} = 0$, which yields our result, that the amplification $A = -R_c / R_e$. This expression also shows the appropriate version for A in the case R_e becomes zero: $A = -R_ch_{fe}/h_{ie}$, an result heavily dependent on electrical parameters of the transistor.

Comments on Use of Negative Feedback in General with Amplifiers. The insight from comparing the voltage regulator to the non-inverting amplifier circuit will now be useful in understanding the value of negative feedback in general use with amplifier design. Specifically, feedback in general compares the result of a “process” to a “goal” and makes “adjustments” based on the difference. Negative feedback makes the adjustments in a way tending to reduce the difference or “error.”

In general-purpose amplifiers, the input is the “reference” to which the output is to be compared. Thus, negative feedback has the property of ***reducing amplifier-induced distortion***.

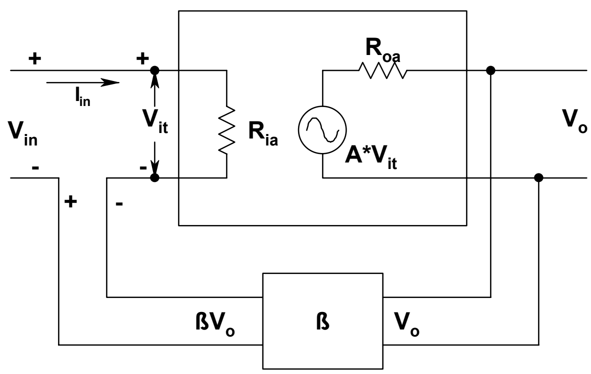
Effect of feedback on Input and Output Resistances.
Our final task will be to examine the correspondence between the characteristics of the basic amplifier, the feedback connection, and the effective input and output resistances of the circuit including feedback. The “standard” circuit from above is copied below for this purpose.

Effective Input Resistance. As before, the approach will be based on the Thevenin concept. For the input resistance, we will “apply” an input voltage, and seek a relation between it and the resulting current so that $R_{in} = V_{in} / I_{in}$. By examination of the circuit, we can see the following:

$$I_{in} = \frac{V_{it}}{R_{ia}} = \frac{V_o}{AR_{ia}} = \frac{GV_{in}}{AR_{ia}} = \frac{1}{(1-A\beta)}\frac{V_{in}}{R_{ia}}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = (1-A\beta)R_{ia}$$

Voltage Feedback Amplifier System



Thus, for ***positive feedback***, where 1-Aβ is less than 1 and may approach zero, the input resistance is ***less than that of the basic amplifier***. In contrast, for negative feedback, where 1-Aβ is positive and greater than 1, the effective input resistance is ***greater than that of the basic amplifier****.* Thus, in terms of a general-purpose voltage amplifier, ***negative feedback enhances*** the input resistance.

Effective Output Resistance. Also as before, our approach will be to use the Thevenin perspective. Specifically, we will seek the ratio of the open-circuit output voltage to the short-circuit output current.

$$V_{out})_{open} = AV_{it} = GV_{in}$$

$$I_{out})_{short} = \frac{AV_{it}}{R_{oa}}$$

However, the case when the output is short-circuited must be examined more carefully since V_o = 0 under those circumstances. Specifically, when V_o = 0, βV_o = 0, also and $V_{it} = V_{in}$. So,

$$R_{out} = \frac{V_{out})_{open}}{I_{out})_{short}} = \frac{GV_{in}}{AV_{it}/R_{oa}} = \frac{AV_{in}/(1-A\beta)}{AV_{in}/R_{oa}} = \frac{R_{oa}}{(1-A\beta)}$$

For positive feedback, where 1-Aβ will be less than 1, and may approach zero, the effective output resistance is ***greater than that of the basic amplifier***. On the other hand, for negative feedback, where 1-Aβ will be greater than 1, the output resistance is ***less than that of the basic amplifier.*** In summary, for a general-purpose voltage amplifier, ***negative feedback improves the effective output resistance***.

Conclusions: Both positive and negative feedback have useful applications. In particular, negative feedback has a wide range of applications in control-system technology. Moreover, negative feedback provides improved characteristics of general-purpose amplifiers: distortion is reduced, the input resistance is increased, and the output resistance is decreased.